DROPLET DISTRIBUTION IN THE BREAKUP OF A LIQUID BY AN ATOMIZER

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A theoretical basis and experimental confirmation are given for the log-normal size distribution of droplets in the total population during the breakup of a liquid by an atomizer.

If the physical model discussed by Levich [1] for the breakup of a liquid during atomization is adopted, whereby droplets break away from the perturbed free surface of the liquid under the dynamic action of the surrounding gas medium, it is found that all waves on that surface with wave numbers satisfying the inequality

$$k \leqslant \frac{\rho_g \bar{u}^2}{\sigma}, \tag{1}$$

are unstable. In other words, an increase in their amplitude can cause droplets to break away from the crest of any such wave. The droplet sizes x in this case correspond in order of magnitude to the wavelengths l_x and are functionally related to them:

$$x = f(l_x).$$

In turn,

$$l_x \sim \frac{1}{k} \sim \frac{\sigma}{\rho_g \bar{u}^2}. \tag{2}$$

At the instant that the droplets break away, the breakup process causes the wavelength of the waves themselves to change to a new value l_{xt} such that $l_{xt} \leqslant l_x$. We noted that the condition for applicability of expression (1) is

$$\frac{\mu_l \bar{u}}{\sigma} \sqrt{\frac{\rho_g}{\rho_l}} \ll 1. \tag{3}$$

This inequality can be expressed in terms of dimensionless criteria:

$$\frac{\mathrm{We}}{\mathrm{Lp}} \ll 1. \tag{4}$$

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The latter condition in a sense complements the requirement

$$\mathrm{We} \geqslant \mathrm{We}_{cr}. \tag{5}$$

For low-viscosity liquids (such as water) and moderate values of $\bar{u}$, as are typical of the operation of most mechanical sprayers and atomizers, inequalities (4) and (5) are almost always upheld.

The stochastic character of l_{xt} causes the formation of a large number of droplets of different sizes and justifies the assumption that the distribution of l_{xt} in the total population obeys statistical laws.

A consideration of the number of l_{xt}, i.e., $N(l_{xt}, t)$, and their distribution at successive times t readily indicates that the assumptions made by Kolmogorov [2] in his theoretical foundation of the log-normal particle-size distribution function for the breakup of solid materials ($t \to \infty$) are also correct in the present problem, and the processes are governed by an identical mechanism.

We show that the size distribution P(x) of the number of drops in the total population is a log-normal function. This means that the end result must be

$$P\{x\} = \lim_{k\to\infty} \frac{\sum_{i=1}^{k} P\{x_i\} n_i}{\sum_{i=1}^{k} n_i}. \tag{6}$$

Considering that $x = f(l_x)$ for the normalized distribution functions $P\{l_{xt}\}$ and $P\ l_{xt}$, we write

$$\lim_{i\to\infty} P\{x_i\} = \lim_{t\to\infty} P\{l_{xt}\} = P\{l_{xT}\}. \tag{7}$$

Then for any arbitrarily small number $\varepsilon > 0$ there is always an N such that for all $P\{x_i\}$, beginning with $i > N$, the following inequality will hold:

$$P\{x_i\} - P\{l_{xT}\} < \varepsilon. \tag{8}$$

We transform the argument of the limit in (6), taking $k \gg N$:

$$\frac{\sum_{i=1}^{k} P\{x_i\} n_i}{\sum_{i=1}^{k} n_i} = \frac{\sum_{i=1}^{k} (P\{l_{xT}\} + P\{x_i\} - P\{l_{xT}\}) n_i}{\sum_{i=1}^{k} n_i} =$$

$$= \frac{P\{l_{xT}\} \sum_{i=1}^{k} n_i}{\sum_{i=1}^{k} n_i} + \frac{\sum_{i=1}^{N} (P\{x_i\} - P\{l_{xT}\}) n_i}{\sum_{i=1}^{k} n_i} +$$

$$+ \frac{\sum_{i=N+1}^{k} (P\{x_i\} - P\{l_{xT}\}) n_i}{\sum_{i=1}^{k} n_i}.$$

Solving the latter term by term, we obtain for $k\to\infty$

$$\frac{P\{l_{xT}\}\sum_{i=1}^{\infty} n_i}{\sum_{i=1}^{\infty} n_i} = P\{l_{xT}\}, \qquad \frac{\sum_{i=1}^{N}(P\{x_i\}-P\{l_{xT}\})\,n_i}{\sum_{i=1}^{\infty} n_i} = 0,$$

$$\frac{\left|\sum_{N+1}^{\infty}(P\{x_i\}-P\{l_{xT}\})\,n_i\right|}{\sum_{i=1}^{\infty} n_i} \leqslant \frac{\sum_{N+1}^{\infty}|(P\{x_i\}-P\{l_{xT}\})\,n_i|}{\sum_{i=1}^{\infty} n_i} < \frac{\sum_{N+1}^{\infty}\varepsilon n_i}{\sum_{i=1}^{\infty} n_i}.$$

Furthermore,

$$\frac{\sum_{N+1}^{\infty}\varepsilon n_i}{\sum_{i=1}^{\infty} n_i} = \frac{\varepsilon\sum_{N+1}^{\infty} n_i}{\sum_{i=1}^{\infty} n_i} + \frac{\varepsilon\sum_{i=1}^{N} n_i}{\sum_{i=1}^{\infty} n_i} - \frac{\varepsilon\sum_{i=1}^{N} n_i}{\sum_{i=1}^{\infty} n_i} = \frac{\varepsilon\sum_{i=1}^{\infty} n_i}{\sum_{i=1}^{\infty} n_i} - \frac{\varepsilon\sum_{i=1}^{N} n_i}{\sum_{i=1}^{\infty} n_i} \leqslant \varepsilon.$$

Finally, $P\{x\} = P\{l_{xT}\}$ as $\varepsilon\to 0$. This means that the distribution of all droplets in the total population of the atomized liquid also obeys a log-normal law. Consequently,

$$P\{d\} = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\lg d}\exp\left[-\frac{(\lg x-\lg\xi)^2}{2\sigma^2}\right]d(\lg x). \tag{9}$$

From the log-normal droplet-size distribution we readily obtain characteristics of the population that are functions of the size (surface area, volume, injection rate, etc.). It is sufficient to analyze the initial moments of order n of the original distribution function.

For example, the surface $F = \pi M_2$ formed in the atomization of a given volume of liquid $V = \frac{\pi}{6}M_3$ is

$$F = \frac{6V}{\xi\exp\left(2.5\,\frac{\sigma^2}{M^2}\right)}, \text{ where } M = \lg e. \tag{10}$$

The fraction of the given droplet characteristics associated with the population of diameters less than d can be obtained from the distribution of the moments themselves. It must be borne in mind here that in real systems there is always a d_{max}, i.e., an upper limit of the distribution function.

Experimental determinations of the distribution of droplets produced in the atomization of liquids by various atomizing devices have been undertaken by many researchers on the basis of a number of postulated empirical and semiempirical dependences, the application of which is restricted to the ad hoc experimental conditions. A recent study [3], in which the most commonly used distributions (Rosin–Rammler, Nukiyama–Tanasawa, normal, log-normal, etc.) are analyzed, shows that for high-pressure spray nozzles the log-normal function best describes the experimental distributions. We have investigated the dispersion characteristic of low-pressure mechanical atomizers.

For a reliable estimation of the experimental parameters ξ and σ it is necessary to take into account the degree of truncation of the total droplet population in samples with respect to d_{min}, which is determined by the particular investigative technique used. The method of estimating the parameters with a known truncation point is taken from [4-6] with the condition $d_{min} < \xi$.

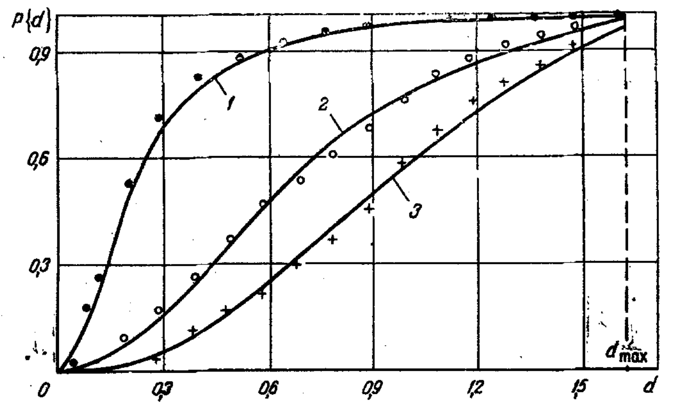

Fig. 1. Distributions of the number (1), surface area (2), and volume (3) of droplets in an atomizer jet; d in mm.

With the foregoing in mind we obtained the parameters of the distribution of water droplets in the jets of three types of mechanical atomizers (one centrifugal and two unit-jet types comprising a combination centrifugal and axial-jet atomizer) at a pressure of 0.5 to 2.5 atm with nozzle diameters of 3 to 8 mm, in application to the misting chambers of artifical climate equipment.

A comparison of the experimental and analytical curves indicates satisfactory agreement of the results. The relative error does not exceed the limits of the experimental procedure [7], as estimated in [8].

Figure 1 gives as an example the distribution functions for the number, surface area, and volume of droplets in the jet of one of the investigated centrifugal atomizers ($A = 0.82$, $d_n = 4.0$ mm, $p = 1.0$ atm, $\xi = 0.2$ mm, $\sigma = 0.35$, $d_{max} = 1.64$ mm).

NOTATION

k is the wave number; ρ_g, ρ_l are the densities of the gas and liquid; σ is the coefficient of surface tension or the variance of the distribution; $\bar{u}$ is the relative velocity of the gas and liquid; x, d is the droplet diameter; l_x, l_{xt}, l_{xT} are the wavelengths on the liquid surface at the breakaway time $t = 0$, after breakaway $t \neq 0$, and the limiting value; μ_l is the dynamic viscosity; $P\{x\}$, $P\{d\}$, $P\{l_{xt}\}$, $P\{l_{xT}\}$ are the distribution functions for the droplet diameters and instability wavelengths on the liquid surface; ξ is the median of the distribution; M_n is the initial moment of order n; F, V are the droplet surface area and volume in a population; A is the geometric parameter of the atomizer; d_n is the atomizer nozzle diameter; p is the pressure in front of the atomizer.

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